IB Calculus SL
Internal Assessment

Calculating Propellant Mass Depending on Orbital Altitude

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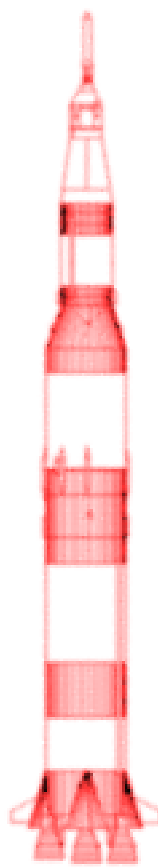
Period 7



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Think of something very expensive, what comes to mind? Maybe you envisioned something like Ferrari or a yacht, but have you ever considered what the price that is required put a satellite into a space? According to Global.com, “It is estimated that a single satellite launch can range in cost from a low of about $50 million to a high of about $400 million.” Additionally, The Union of Concerned Scientists reports that there are around 1,459 active satellites orbiting earth as of 12/31/16. If every one of these active satellites costed the minimum price of a satellite, that comes out to just under $73 billion. Now while many of these satellites are both way below and way above this $50 million price tag, it’s easy to see the immense cost satellites carry.

Therefore, my aim is to calculate, mathematically, the most ideal orbital altitude for a satellite over a 10 year period with respect to two factors. First, the amount of fuel (expense) it would require to launch the satellite into an orbit at that altitude. Second, the air resistance that would act against the satellite, which could possibly require more fuel be used to reaccelerate the satellite to the required orbital speed so that it can maintain its altitude. By “most ideal orbit,” I am simply trying to find the altitude at which the least amount of fuel is required to both get the satellite there and keep it at that altitude.

I am involved with P.S.A.S (the “Portland State Aerospace Society”), in building and launching rockets and I have begun talking to some of my own classmates about beginning a rocketry club of our own. We are already beginning to create some computer models of the rockets we plan to build. These programs have spiked my interest in orbital physics and aided in my selection of this topic.

To create my equation, I will need to create an equation that calculates that amount of fuel required, which for this paper will be defined as “**Y**,” based on the orbital altitude, which will be defined as “**X**.” To create this equation, I will need to be able to find a way to calculate the fuel required and the air resistance encountered based on the altitude and the altitude alone, otherwise there will be too many variables to solve and graph for. In other words, my final equation for calculating the most ideal orbit must have only two variables: orbital altitude (**X**) and weight of fuel required (**Y**).

Now, because of the restraints of my abilities and my knowledge on orbital mechanics, I will be making a number of different assumptions. Because different spacecraft are affected differently by orbital and fluid mechanics, the spacecraft being examined in this IA must be defined. For both conventional and computational purposes, my calculations will be based around the Falcon 9 Full Thrust rocket, a rocket designed by SpaceX. This rocket has a dry mass of 26,200 kg (the first stage weighing 22,200 kg, and the second stage weighing 4,000 kg) and its fairing has a nose cone that is 13 meters long and 5.2 meters in diameter, with a slant length of 14m. The specific impulse of the first stage is 282 seconds, and while this is the value at sea level, I am not expecting to use this stage in space. The specific impulse of the second stage is 348 seconds. Any spacecraft could be used in place of what I am using, it would just require a change in some of the constants in the equation. I will be assuming that this spacecraft will have perfectly circular orbits and that their total change in velocity is the final velocity they reach at their intended orbit. I will be measuring mass in kilograms, length in meters and time in seconds. I am only focusing on the behavior of the rocket in orbit, so the fuel and calculations for in atmosphere flying will not be included in my calculation. I will only include the speed at which the rocket is moving when it reaches orbit. For ascension, I am assuming mass for both stages (though in reality, the first stage is released part way through the ascension), and for everything that has to do with the rocket already in orbit, I will use the weight of just the second stage. I am assuming that the acceleration at the surface of the earth is $9.81 m/s^2$.

Defining the Structure of My Internal Assessment:

My aim is to calculate, mathematically, the most ideal orbital altitude for a satellite over a 10 year period. My final equation must acknowledge all the parts of a rocket's flight to be able to calculate the altitude at which fuel consumption is the least. These parts include:

1. The vertical ascent that carries the rocket to a certain altitude
2. The circularization burn that puts the rocket in a circular orbit
3. The burns that are required to counter the force of air resistance on the rocket

These 3 parts will be the 3 big sections of my paper and will contribute to creating the final equation that will determine the most ideal altitude. I am planning to make my final equation look as such:

Y = (fuel for ascent and circularization with respect to X)+(fuel for air resistance with respect to X)

Calculating the Equation for Fuel Consumption at a Certain Altitude:

This part of the equation has two parts within itself. I will find the velocity it takes to get the rocket to the altitude, and the the velocity it takes to then create a circular orbit. I will start with the velocity it takes to get the rocket into a circular orbit. To find the required velocity to create a circular orbit, I will be using the Tsiolkovsky rocket equation, which describes the motion of vehicles that follow the basic principle of a rocket. This equation allows me to obtain the weight of the fuel required ($\mathbf{m_f}$) with respect to the change in velocity that is required to achieve a certain orbital altitude ($\mathbf{\Delta V_o}$). The equation looks like this:

$$\Delta v_o = v_e \ln \frac{m_0}{m_f}$$

Where:

$\mathbf{\Delta V_o}$ = Total change in velocity (in other words, the final orbital velocity of the spacecraft).

$\mathbf{V_e}$ = The effective exhaust velocity (meters per second or "m/s")

ln = natural log

$\mathbf{m_f}$ = The mass of the rocket without propellant, also known as "dry mass" (kilograms, or more commonly notated as "kg")

$\mathbf{m_o}$ = The initial total mass of the rocket (in other words, $\mathbf{m_f}$ + the weight of the propellant) (kg)

In this equation, I am trying to find the weight of the propellant with respect to the altitude. The variable $\mathbf{m_f}$ holds my propellant weight and $\mathbf{\Delta V_o}$ will be the variable for the change in velocity I need to reach to achieve a certain orbit (I will calculate this velocity using both altitude and orbital period). Therefore, the equation must be rearranged to fit this:

$$\Delta V_o/V_e = \ln(m_o/m_f)$$

$$e^{(\Delta Vo/Ve)} = m_o/m_f$$

$$m_o = m_f \bullet e^{(\Delta Vo/Ve)}$$

And because I know that $\mathbf{m_o}$ = $\mathbf{m_f}$ + **Y** ("**Y**" being the weight of the fuel on its own), I can further edit the equation to make:

$$Y + m_f = m_f \bullet e^{(\Delta Vo/Ve)}$$

$$Y = m_f \bullet e^{(\Delta Vo/Ve)} - m_f$$

I know that m_o = 4,000 kg from my assumptions, and this is because I am assuming that only the second stage of the Falcon 9 rocket will be trying to reach orbit. I also know the value of "e," because e is Euler's number, which has a value of 2.7182… and so on. I only need to figure out the values of $\mathbf{V_e}$ and ΔV to complete this part of the equation. $\mathbf{V_e}$ is the effective exhaust velocity, and through some research, I found an equation to find this value on the wikipedia page for "Specific impulse." The equation is:

$$v_e = g_0 I_{sp}$$

Where:

$\mathbf{V_e}$ = The effective exhaust velocity (m/s)

$\mathbf{g_0}$ = acceleration due to gravity at the Earth's surface (m/s^2)

$\mathbf{I_{sp}}$ = The specific impulse, measured in seconds.

It is a widely known fact that the acceleration at the earth's surface is 9.81m/s^2, and that is the value I need for $\mathbf{g_0}$. According to the wikipedia page on the Falcon 9 Full Thrust rocket, its combined specific impulse of both stages is 630 seconds. Inputting those values into my equation gets me:

$$\mathbf{Ve = 9.81m/s^2(630s)}$$

$$\mathbf{Ve = 6180.3\ m/s}$$

Continuing on, $\mathbf{\Delta V_o}$ is the part of this equation that will contain my X value. Since "X" in this case is altitude, I need to create an equation for $\mathbf{\Delta V_o}$ with respect for "X" that I can replace $\mathbf{\Delta V_o}$ with within the larger equation. It should be mentioned that there is an equation that fits these requirements very well, however, in this case, I am going to use a more mathematical approach and use arc lengths to find the $\mathbf{\Delta V_o}$ for the rocket. Instead of using the whole 360° of what is a circular orbit which is just the circumference, I will stick with 45° and reduce my orbital period value by an eighth (because 45/360 is 1/8) to fit the equation. Since my $\mathbf{\Delta V_o}$ is the velocity of my orbit, the equation for velocity will just be:

ΔV_o = meters/seconds = (The arc length of the orbit)/(The time it takes to travel that length)

To find the arc-length of the orbit, I will use this equation from the IB Mathematics SL formula booklet:

$$\textbf{Arc length (m)} = \boldsymbol{\theta} \bullet (\pi/180) \bullet \mathbf{r}$$

Where:

$\boldsymbol{\theta}$ = My angle measure(**45°**)

r = the total radius (which is the radius of the earth added to the altitude)

Searching quickly on google, I found that the radius of the earth is 6,371,000 meters. Therefore, I can rewrite the arc length equations as:

$$\textbf{Arc length (m)} = \mathbf{45 \bullet (\pi/180) \bullet (6{,}371{,}000m + X)}$$

$$\textbf{Arc length (m)} = \mathbf{(\pi/4) \bullet (6{,}371{,}000m + X)}$$

To find time, I found the orbital period equation on the physics classroom website, which is:

$$\mathbf{T = \sqrt{[(4 \bullet \pi^2 \bullet R^3) / (G \bullet Mc)]}}$$

Where:

T = Orbital period (seconds)

R = the radius of the orbit (meters, more commonly notated as "m")

G = the gravitational constant (6.673 x 10^{-11} N m^2/kg^2)

Mc = The mass of the celestial body that the satellite is orbiting (kg)

I already know that the radius of the orbit is 6,371,000 meters added to the altitude(**X**). Another quick search on google and I find that the mass of earth is 5.98 x 10^{24} kg. The gravitational constant is just that, a constant and its value is given above. I am going to put a ⅛ into the equation to compensate for the fact that I am not finding the whole orbital period, only 45° of it (which is an eighth of 360°). Now that I have all my constants and variables converted to values, I can rewrite my orbital period equation as:

$$\mathbf{T = \tfrac{1}{8} \cdot \{\sqrt{\left[\dfrac{(4 \cdot \pi^2 \cdot (6{,}371{,}000m + X)^3)}{(6.673 \cdot 10^{-11}\ N\ m^2/kg^2) \cdot (5.98 \cdot 10^{24}\ kg)}\right]}\}}$$

$$\mathbf{T = \tfrac{1}{8} \cdot \{\sqrt{\left[\dfrac{(4 \cdot \pi^2 \cdot (6{,}371{,}000m + X)^3)}{(3.99 \cdot 10^{14}\ N\ m^2/kg)}\right]}\}}$$

Now that I have all the parts of $\mathbf{\Delta V_o}$, I can compile it. This is what the $\mathbf{\Delta V_o}$ equation looks like:

$$\mathbf{\Delta V_o = \dfrac{Arc\ Length}{Orbital\ Period} = \dfrac{(\pi/4) \cdot (6{,}371{,}000m + X)}{\tfrac{1}{8} \cdot \{\sqrt{[(4 \cdot \pi^2 \cdot (6{,}371{,}000m + X)^3) / (3.99 \cdot 10^{14})]}\}}}$$

$$\boxed{\mathbf{\Delta V_o = \dfrac{2 \cdot \pi \cdot (6{,}371{,}000m + X)}{\sqrt{[(4 \cdot \pi^2 \cdot (6{,}371{,}000m + X)^3) / (3.99 \cdot 10^{14}\ N\ m^2/kg)]}}}}$$

And now that all my values for my original equation are defined, I can finish my original equation with those values:

$$\mathbf{Y = m_f \cdot e^{(\Delta Vo/Ve)} - m_f = (4000kg \cdot e^{(\Delta Vo/6180.3\ m/s)}) - 4000}$$

$$\boxed{\mathbf{Y = (4000kg \cdot e^{(\Delta Vo/6180.3\ m/s)}) - 4000kg}}$$

For now, I am going to leave $\mathbf{\Delta V_o}$ as it is because I don't want to write out the full equation in that tiny area. This is the equation for finding the amount of fuel in kilograms needed to get the rocket into a circular orbit at a certain altitude, if I am just accounting for altitude.

<u>Calculating the Equation for Fuel Consumption to Reach a Specified Altitude</u>

To find the velocity it takes to bring the rocket to a specific orbit, considering that in this world, air resistance doesn't make a difference, I found that I can use a kinematic equation to find the required velocity it takes to bring something to a specific altitude, this kinematic equation is:

$$\mathbf{V_f^2 = V_i^2 + (2 \cdot a \cdot X)}$$

<u>Where:</u>

$\mathbf{V_f}$ = Final velocity (which is the peak of the ascent, therefore this value is 0 m/s) (m/s)

$\mathbf{V_i}$ = Initial velocity (basically my total velocity or ΔV_r, this is the speed the rocket must achieve to reach the intended orbit) (m/s)

a = acceleration due to gravity (-9.8m/s^2)

X = distance travelled (m)

I am going to need to rearrange this equation so I can solve for **Vi** because **Vi** is the initial velocity, the velocity I begin with. If my final velocity is zero, and I'm slowing down, then **Vi** is the velocity I need to get to the height where my final velocity equals 0. It gives me the velocity I need in one place, instead of over a spread of time:

$$\mathbf{V_i = \sqrt{(V_f^2 - (2 \bullet a \bullet X))}}$$

Which, when some of the constants are replaced with values (and this is where I rename $\mathbf{V_i}$):

$$\mathbf{\Delta V_r = \sqrt{(0^2 - (2 \bullet -9.8 \bullet X))}}$$

$$\mathbf{\Delta V_r = \sqrt{(19.6 \bullet X)}}$$

And I can now put this back into the Tsiolkovsky rocket equation, to get a total fuel mass for this:

$$\mathbf{Y = m_f \bullet e^{(\Delta Vr/Ve)} - m_f}$$

$$\boxed{\mathbf{Y = 26200kg \bullet e^{(\sqrt{(19.6 \bullet X)}/6180.3\ m/s)} - 26200kg}}$$

This equation, and the equation before this, are the equations that help us find the required fuel for the rocket to achieve an altitude at a specific orbit.

Calculating the Equation for Fuel Consumption with Respect to Air Resistance:

To calculate air resistance, I will use this accepted equation:

$$F_a = kv^2 = \frac{\rho C_D A}{2} v_o^2$$

Where:

$\mathbf{F_a}$ = The force of air resistance (Newtons)

k = A constant that includes the traits of the atmosphere

$\mathbf{V_o}$ = orbital velocity (m/s)

p = The density of the air that the craft is moving through (kg/m^3)

$\mathbf{C_D}$ = The drag coefficient, a value that changes based on the shape and size of a craft

A = Area of craft that is in contact with the air (m^2)

I know that the the velocity of the craft is equal to the equation figured out in the last section. To figure out the drag coefficient, I merely needed to know the shape of craft that was in contact with the atmosphere, which is the Falcon 9 payload fairing, a nose cone. According to the wikipedia page for this very topic, the drag coefficient of a cone is 0.5. To find "**A**," I need to find the area of the cone that is in contact with the air, which can be found by using the cone surface area equation:

$$\mathbf{A = \pi r^2 + \pi rs}$$

Where:

r = radius (m)

S = slant length (m)

And I can pull the different inputs for this equation from my assumptions:

$$\mathbf{A = \pi \bullet (2.6m)^2 + \pi \bullet (2.6m) \bullet (14m)}$$

$$\mathbf{A = 43.16\pi}$$

Now the final variable I need to find is air density. This next section shows how to calculate that.

Calculating Air Density for the Equation:

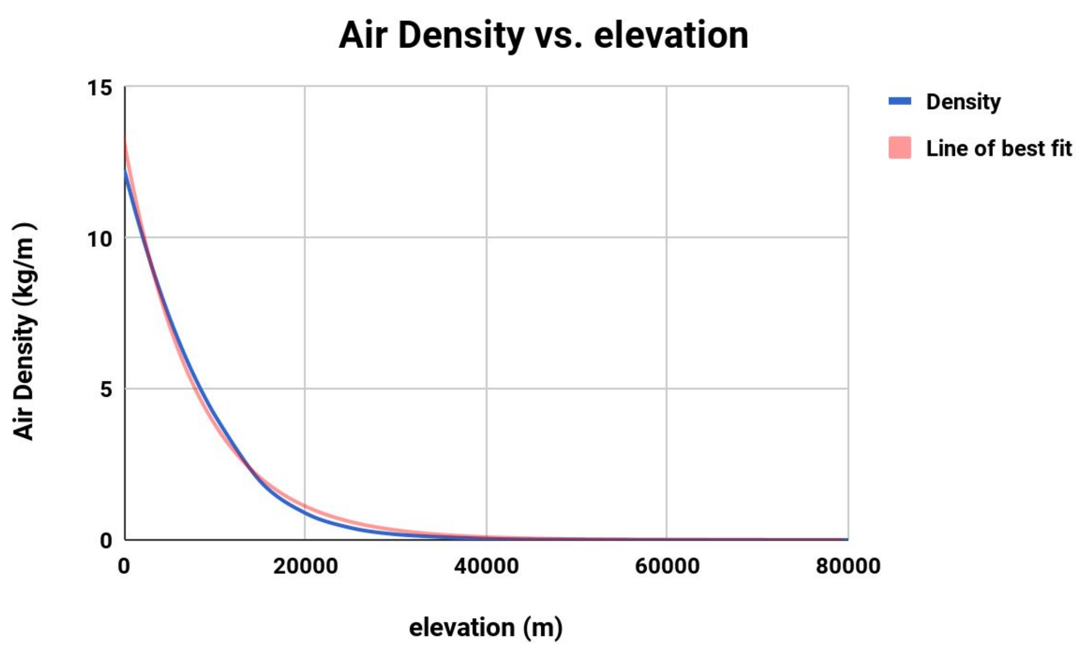
The next step in finding air resistance is calculating air density. To find air density, I first attempted to use a physics equation but it didn't work. Then, I decided to use already known data to create a mathematical equation that would work just the same. I found some online data from The Engineering Toolbox that had multiple air density values for different altitudes. The data from that site looked like this:

Elevation (m)	Air Density (Kg/m^3)
0	1.225
1000	1.112
2000	1.007
3000	0.9093
5000	0.7364

Elevation (m)	Air Density (Kg/m^3)
10000	0.4135
15000	0.1948
20000	0.08891
25000	0.04008
30000	0.01841

Elevation (m)	Air Density (Kg/m^3)
40000	0.003996
50000	0.001027
60000	0.0003097
70000	0.00008283
80000	0.00001846

Which, when graphed, looks like this:



On this graph, the line of best fit (the orange line) is the equation that describes the equation of the graph with respect to elevation. In other words, that is the equation for the air density with respect to altitude(**X**). Therefore, my equation for air density is:

$$P = 1.32 \cdot e^{(-0.000123 \cdot X)}$$

Calculating the Equation for Fuel Consumption with Respect to Air Resistance Continued:

Finding air density was the last thing I needed to do to get all my variables for the air resistance equation. Now, I can compile everything into a single expression. ΔV_o would just be the value from the ΔV_o equation from before. I'm using ΔV_o in this calculation because this equation is talking about air resistance in orbit, and so I'm pulling the velocity equation for orbital velocity, air resistance during ascent is not being factored in here:

$$F_a = \frac{P \bullet C_d \bullet A \bullet (\Delta V)^2}{2} = \frac{1.32 \bullet e^{(-0.000123 \bullet X)} \bullet 0.5 \bullet 43.16\pi \bullet (\Delta V_o)^2}{2}$$

$$F_a = 14.2428\pi \bullet e^{(-0.000123 \bullet X)} \bullet (\Delta V)^2$$

To then incorporate this into my final equation, I need to convert this into another unit, one that I can derive a change of velocity from that I can then get a fuel mass from. I'll start this process by incorporating my air resistance into the commonly used force equation:

$$F = M \bullet A$$

Which can be rewritten as:

$$A = \frac{F}{M} = \frac{Fa}{M} = \frac{14.2428\pi \bullet e^{(-0.000123 \bullet X)} \bullet (\Delta V)^2}{M}$$

Where:

A = Acceleration (m/s^2)
F = Total Force (Newtons)
M = Mass (kg)
Fa = Force from air resistance (Newtons)

I can find the change in velocity by finding out the acceleration of the spacecraft over a set period of time. I am mapping the orbit of the spacecraft over 10 years, which is 315360000 seconds. Without a good knowledge of the rocket's mass during this time, because the mass of the rocket would be the dry mass along with remaining fuel, which would be always changing. I'm going to replace "**M**" with the dry mass. The equation for **ΔV** due to air resistance (which I will refer to as **ΔVa**) will be:

$$\Delta Va = A \bullet t$$

Where:

ΔVa = Change in velocity due to air resistance (m/s)
A = Acceleration (m/s^2)
t = Total change in time (s)

Replacing acceleration with my equation and time with 315360000 seconds (equivalent to 10 years) and mass with 4000 kg, my final equation for change in velocity looks like this:

$$\mathbf{\Delta Va = A \cdot t = \frac{14.2428\pi \cdot e^{(-0.000123 \cdot X)} \cdot (\Delta V)^2}{4000kg} \cdot 315360000s}$$

$$\boxed{\mathbf{\Delta Va = \frac{4491609408\pi \cdot e^{(-0.000123 \cdot X)} \cdot (\Delta V)^2}{4000kg}}}$$

And I can now reuse the Tsiolkovsky rocket equation to get a weight value of the fuel that will be needed to maintain the rocket's orbit:

$$\Delta v = v_e \ln \frac{m_0}{m_f}$$

Or, as simplified before:

$$\mathbf{Y = m_f \cdot e^{(\Delta Va/Ve)} - m_f}$$

As given in my assumptions, the dry mass (**m_o**) of the second stage of the rocket is 4,000 kg. To find **V_e**, I just have to use the same equation as above, but my specific impulse will be much shorter, I stated that in my assumptions that the specific impulse of the second stage (the stage in space) is 348 seconds. I am only using information for the second stage because this is the stage that will be orbiting around the earth at this point. I'm assuming that the first stage would have been ejected long before this. With this information, I can solve for **V_e**:

$$v_e = g_0 I_{sp}$$

<u>Where:</u>

V_e = The effective exhaust velocity (m/s)

g_0 = acceleration due to gravity at the Earth's surface (m/s^2)

I_{sp} = The specific impulse (seconds)

$$\mathbf{V_e = 9.81 m/s^2 \times 348s}$$

$$\mathbf{V_e = 3413.88\ m/s}$$

And then compiling everything into the Tsiolkovsky rocket equation gives me:

$$\mathbf{Y = m_f \cdot e^{(\Delta V/Ve)} - m_f = \boxed{4000kg \cdot e^{(\Delta Va/3413.88m/s)}) - 4000kg}}$$

<u>The Final Equation:</u>

Now that I have each of the fuel weight equations to both get us to a certain altitude and keep us there, I can compile each of these equations together to make a final equation equation that will tell me what altitude would be most ideal to launch a satellite to that would require the least amount of fuel. This equation would be the sum of all three equations, since the total fuel would be the fuel that the rocket uses to achieve each of these different parts. The final equations would look like this:

$$\mathbf{Y = [(4000 \cdot e^{(\Delta Vo/6180.3)}) - 4000] + [26200 \cdot e^{(\sqrt{(19.6 \cdot X)})/6180.3)} - 26200] + [4000 \cdot e^{(\Delta Va/3413.88)} - 4000]}$$

Which can be further simplified to...

$$Y = (4000 \cdot e^{(\Delta V_o/6180.3)}) + (26200 \cdot e^{(\sqrt{(19.6 \cdot X)}/6180.3)}) + (4000 \cdot e^{(\Delta Va/3413.88)}) - 34200$$

Where:

Y = The total weight of fuel in Kilograms

X = Altitude in meters

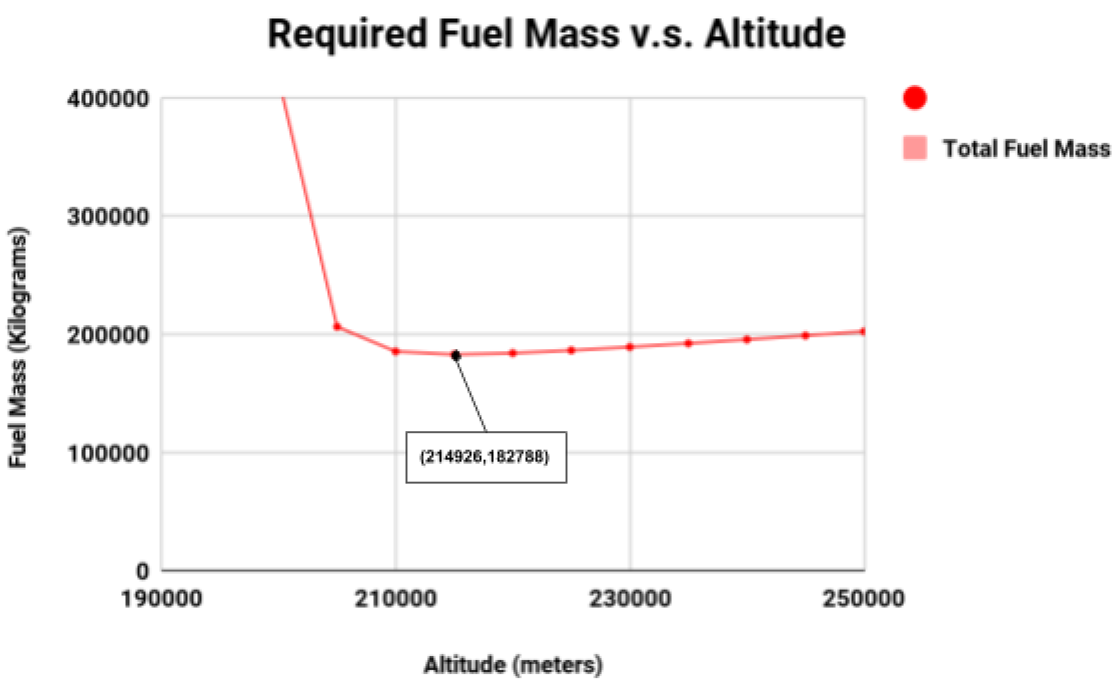
e = Euler's number

$$\Delta V_o = \frac{2 \cdot \pi \cdot (6{,}371{,}000 + X)}{\text{SQRT } [(4 \cdot \pi^2 \cdot (6{,}371{,}000 + X)^3) / (3.99 \cdot 10^{14})]}$$

and

$$\Delta Va = \frac{4491609408\pi \cdot e^{(-0.000123 \cdot X)} \cdot (\Delta V_o^{\,2})}{4000\text{kg}}$$

When graphed, the equation makes a parabola that describes the relationship between the fuel mass and the altitude of orbit/air resistance. The line graphed looks like this:



The lowest point on this graph tells me the altitude at which the lowest amount of fuel is required to maintain an orbit there for 10 years. The minimum point on the parabola is (214926,182788) and this tells me that the altitude at which the least amount of fuel is needed to maintain an orbit for 10 years with the Falcon 9 Full Thrust rocket is **214,926 meters**. It may seem on the graph like this is the minimum value in relation to other points I found in excel but I did confirm that this was the absolute minimum on my Ti-84 graphing calculator.

Verifying my Final Answer Using Derivatives:

Now that I have found a point on the graph, I can check my answer my finding the derivative at the minimum point to see if the slope there is zero. If it is, then that confirms that **(214926,182788)** is in fact the minimum of my graph. Due to the sheer size of my final equation, I won't be solving it by hand. The derivative would look like this:

$$\frac{dY}{dX}\left[\mathbf{Y = (4000 \cdot e^{(\Delta V_o/6180.3)}) + (26200 \cdot e^{(\sqrt{(19.6 \cdot X)}/6180.3)}) + (4000 \cdot e^{(\Delta V_a/3413.88)}) - 34200} \right]\Bigg|_{214926}$$

When evaluated using a calculator, I can confirm that the slope of my equation at **(214926,182788)** is zero, which confirms my answer that the altitude at which the minimum mass of fuel is needed to reach it is in fact **214926** meters.

Final Reflection and Conclusion:

What has this equation found for me? This equation has found a relationship between the altitude of an orbit and the time it is expected to fly to the mass of the propellant needed to get it there and keep it there. In making this equation, I have taken different equations from different areas of physics, with all different kinds of variables, and I have manipulated these equations to get the relationship between two and only two variables, by either cancelling out, or making them constants, which is what I needed to discover a relationship. I was able to do this by researching an abundance of physics equations in a numerous amount of topics, to find different equations that had specific variables that I needed. The Tsiolkovsky rocket equation gave me the $\mathbf{M_o}$ value, the value that contained the mass of fuel I needed. To get this value, I needed to know the $\mathbf{\Delta V}$ of the rocket, and from here the chain began of finding equations that finally made a link between $\mathbf{M_o}$ and the altitude (**X**) of the rocket.

What can this information and equation do for us? what is a purpose for it in real life? Well, this equation is the basis for figuring out how to launch satellites into space more efficiently. My equation allows anyone who uses it, to be able to calculate the same thing I did, the most ideal altitude to launch something to, that requires the least amount of fuel. This equation can be changed to fit any spacecraft known to man, all it takes is changing some of the constants ($\mathbf{m_o}$, $\mathbf{m_f}$, $\mathbf{I_{sp}}$, $\mathbf{V_i}$, $\mathbf{C_d}$, **etc.**). This equation is also applicable to other celestial bodies too, since the equations within the final equation contain constants that are based on the body of influence ($\mathbf{g_o}$, **G**, $\mathbf{M_c}$, **etc.**). Equations like mine are paramount to rocket science and calculating where a rocket will end up and how. While my equation is somewhat crude in terms of how actual physics is supposed to work, it is a good example of the kind of math and science that rocket scientists use to try to figure out the behavior of a rocket in space.

What are some things that I have learned and things that you can learn from this? I myself have a background in physics and math from classes and extracurriculars that I have been a part of but this is the first time that I have manipulated equations in such a large and extensive fashion. Before this, physics problems gave all the variables and constants except for one, maybe two of them. In this case, except for constants that have a universal value (**e**, $\mathbf{m_f}$, $\mathbf{g_o}$, **G, etc.)**, I had to solve for the variables myself, using a plethora of equations. Orbital mechanics is not something that my teachers and mentors knew much about and so I had to teach myself many of the physics topics that I used in this paper. Another thing I learned as part of this internal assessment is how air density, air resistance, and the fuel required to reach an altitude, increase in relation to altitude. I had thought, before writing this essay, that the mass of fuel required to get to an altitude was directly proportionate to the increase in altitude, but I learned that the fuel required actually follows a more exponential curve, which was an interesting discovery for me.

My own major difficulties in writing this mainly came from solving the equations. When I was first writing this paper, I rearranged the Tsiolkovsky rocket equation incorrectly, and so when I first tested different altitude values in the equation, I got very outlandish numbers, which set me back a couple days. I discovered my error shortly later, somewhat by chance, because I was reexamining the Tsiolkovsky rocket equation for a different part of the paper. Related to that, another difficulty I had was correctly solving and using the air resistance formula. The force of air resistance formula required another equation within itself, thereby making it very hard to solve right. Initially, I had difficulty confirming the validity of the equations I was making and the answers I was finding. I solved this by creating a google spreadsheet, placing a different equation from my internal assessment in each of the columns. This became crucial to my ability to progress in my paper and have realistic answers because I could look at all my data in a row and see the rate of change of different aspects of my rocket's ascent at different altitudes.

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